3D Rigid Body Simulation

Tuesday, November 16, 2021 2:10 PM

3D Object Simulation:

→ simulate rotational & translation dynamics

Newton-euler eqns of motion

translation movement: $ma = \sum_i f_i$

where m = mass, e.g resistance to movement, weight

a = acceleration

$\sum_i f_i$ = sum of external forces

note: v is velocity

$\|v\|$ is speed

$a = \dot{v} = \ddot{x}$

dx/dt (velocity)

d^2x/dt (position)

Rotational Movement: $I\dot{\omega} + \omega \times I\omega = \sum_i \tau_i$ ← sum of torques

where I = moment of inertia, e.g. resistance to turning (analogous to mass). A matrix.

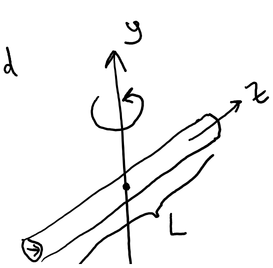
$\dot{\omega}$ = angular acceleration, ω = angular velocity } vectors

Note: ω represents the axis of rotation

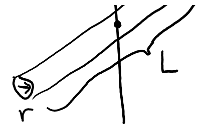
$\|\omega\|$ speed of rotation

Inertia is a matrix: $I \in \mathbb{R}^{3\times 3}$

$\boxed{EX}$ Rod



$$\begin{pmatrix} \frac{1}{4}m(r^2 + \frac{1}{3}L^2) & 0 & 0 \\ 0 & \frac{1}{4}M(r^2 + \frac{1}{3}L^2) & 0 \\ 0 & 0 & \frac{1}{2}Mr^2 \end{pmatrix}$$



For simulation, each object needs the following state

position $(x, y, z)^T$: location of the object

com $(x, y, z)^T$: center of mass, location where sum of all mass in the object sum to zero

→ if you "push" (apply linear force) the com, the body will translate but not rotate

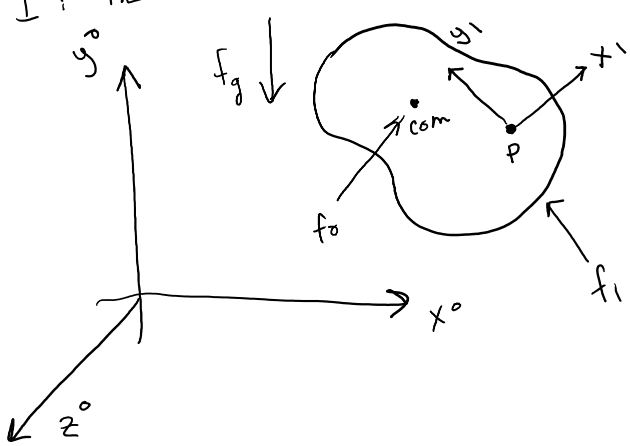
orientation $R \in \mathbb{R}^4$ (quat) or $R \in \mathbb{R}^{3\times3}$ (matrix) : orientation of the object

linear velocity $\in \mathbb{R}^3$

angular velocity $\in \mathbb{R}^3$

mass : weight $\in \mathbb{R}$

I : inertia



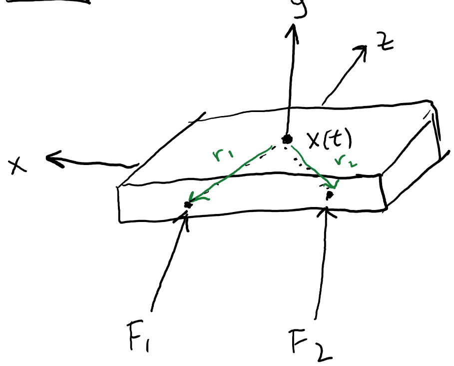
User will set forces (& maybe torques)

Net Force $F = \sum f_i$

Net torque $\tau = \sum (r_i \times f_i) = \sum \tau_i$

Simulator computes accelerations, velocities, positions

$\boxed{\text{EX}}$ Pushing a block w/ equal forces



Let $x(t)$ be the position of the block at time t

The position & com are the same

Suppose $F_1 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$ pushes the box at $\begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix}$ w.r.t the box's frame

Suppose $F_2 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$ pushes the box at $\begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$

What do you expect to happen?

What do you expect to happen?

$\longrightarrow$ box moves forward

$\longrightarrow$ no spinning

<u>Check</u>: Compute acceleration & net torque

$$a = \frac{\sum f_i}{m} = \frac{1}{m}\left(F_1 + F_2\right) = \begin{pmatrix} 0 \\ 0 \\ 2f/m \end{pmatrix}$$

$$\text{net torque} = \sum_{i=1}^{2} \tau_i = \sum_{i=1}^{2} r_i \times F_i = r_1 \times F_1 + r_2 \times F_2$$

$$\Rightarrow \sum \tau_i = \left[\begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

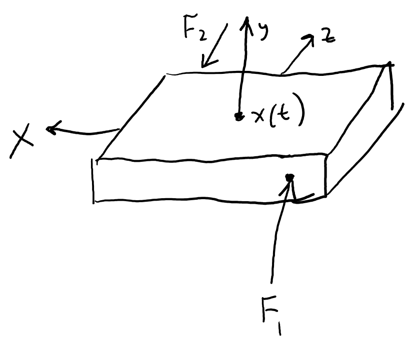
$$= \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \longleftarrow \text{vectors are co-linear} \rightarrow \text{cross product is } \phi$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\boxed{\text{EX}}$ Same box as before, but w/ equal & opposite forces, e.g.

$F_1 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$ acts on the box at point $\begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix}$

$F_2 = \begin{pmatrix} 0 \\ 0 \\ -f \end{pmatrix}$ acts on the box at point $\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$



$$\text{Net force} = \sum_{i=1}^{2} F_i = F_1 + F_2 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Net Torque} = \sum_{i=1}^{2} \tau_i = (r_1 \times F_1) + (r_2 \times F_2) = \left[\begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix} \times F_1\right] + \left[\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} \times F_2\right]$$

Net Torque $= \sum_{i=1}$

$$= \begin{pmatrix} 0 \\ 6f \\ 0 \end{pmatrix}$$