Transforms (cont.)

4x4 matrix transforms are called homogeneous transformations because the operate on homogeneous coordinates

Notes: homogeneous transforms can be represented as 2x2 block matrics

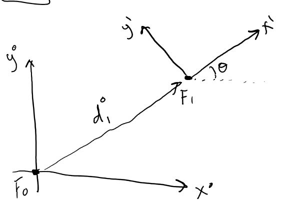
$$\left[\begin{array}{c|c} R & d \\ \hline 0 & 1 \end{array}\right]$$

homogeneous coordinates can be represented as 2x1 block matrices

points $\left[\begin{array}{c} p \\ \hline 1 \end{array}\right]$ and vector $\left[\begin{array}{c} v \\ \hline 0 \end{array}\right]$

We can combine these block representations the same as regular matrix + vector arithmetic

$\boxed{\text{Ex}}$ Consider our frame of reference from last class




The transformation $T_1^0$ which converts coordinates from frame 1 ($F_1$) to frame $\phi$ ($F_0$) is

$$T_1^0 = \left[\begin{array}{c|c} R_1^0 & d_1^0 \\ \hline 0 & 1 \end{array}\right]$$

Suppose $R_1^0 = R_Z(45)$ and $d_1^0 = (5,2,0)^T$.
What is the location of the origin of $F_1$ w.r.t. $F_0$?

Aside: $F_1$ is an example of a local coordinate system

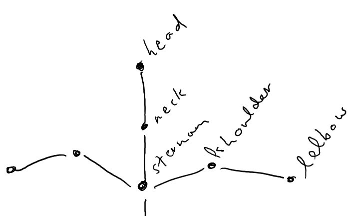
Let $p^1 = (0,0,0)^T$, then $p^0 = T_1^0 p^1$

$$= \left[\begin{array}{c|c} R_1^0 & d_1^0 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c} p^1 \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} R_1^0 p^1 + d_1^0 \\ \hline 0 \cdot p^1 + 1 \end{array}\right] = \left[\begin{array}{c} R_1^0 p^1 + d_1^0 \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} R_Z(45)\begin{pmatrix}0\\0\\0\end{pmatrix} + \begin{pmatrix}5\\2\\0\end{pmatrix} \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} \begin{pmatrix}5\\2\\0\end{pmatrix} \\ \hline 1 \end{array}\right]$$

What is the position of $p^1 = (1,0,0)^T$ w.r.t. $F_0$?

$$p^0 = T_1^0 p^1 = \left[\begin{array}{c} R_1^0 p^1 + d_1^0 \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} R_Z(45)\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}5\\2\\0\end{pmatrix} \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} \begin{pmatrix}\sqrt{2}/2\\ \sqrt{2}/2\\ 0\end{pmatrix} + \begin{pmatrix}5\\2\\0\end{pmatrix} \\ \hline 1 \end{array}\right] = \begin{bmatrix} \frac{\sqrt{2}}{2} + 5 \\ \sqrt{2}/2 + 2 \\ 0 \\ 1 \end{bmatrix}$$

## Articulated Characters

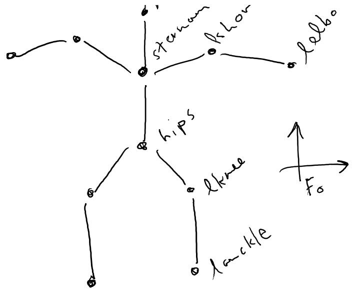



→ Represented as a hierarchy of joints (aka bones)

→ hierarchy is called a skeleton

→ root of the hierarchy is usu. the hips

→ the root transform is usu. w.r.t. to the [illegible] fram




→ root of the

→ the root transform is usu. w.r.t. to the world transform

→ each joint stores a transform w.r.t. to its parent

→ joints with no children are called <u>end effectors</u>

<u>Degrees of freedom (DOF)</u>: the "# ways" an object can move

$\boxed{EX}$ the "hips" have 6 DOF: translation in XYZ
rotation around X Y Z axes

$\boxed{EX}$ the "lknee" might have 3 DOF: rotation around XYZ

<u>Joints</u>: Several types of joints

1. Ball joint (ex. shoulder) ← all joints in our class are these (3 DOF, rotation around XYZ)
2. Hinge joint (ex. elbow) ← 1 DOF joint (rotational)
3. Prismatic joint (ex. selfie stick) ← 1 DOF (translational)

Implementation details (base code):

Skeleton maintains tree of joints

Each joint stores
<u>2 transforms</u>: (1) local2parent ($F_i^j$): converts from the joint's frame to its parent frame

(2) local2global ($F_i^0$) converts from the joint's frame to the global frame

name (ex. "hips")
id (0, 1, 2, ..., numJoints − 1)

ptr to parent
list of children ptrs.

<u>Kinematics</u>: refers to the motion of objects over time

<u>Forward Kinematics</u>: Process of computing global positions of each joint given the current state of the skeleton.

The current state of a skeleton is called a "pose".

Idea: A pose contains a value for every DOF in the character.

( + the joints)

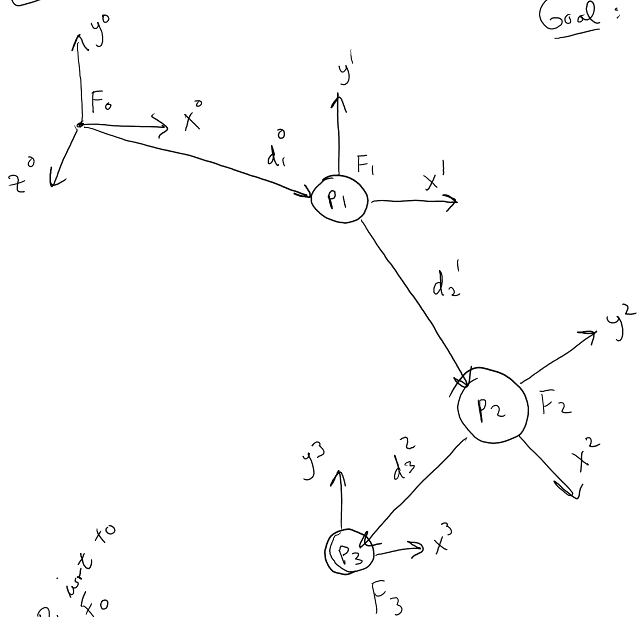
$\boxed{EX}$ The above skeleton has $6 + 11*3$ DOFs (root , other j )

Therefore, our pose have 39 values in it

$$\Theta = [\underbrace{root_x, root_y, root_z}_{\text{root translation}}, \underbrace{root\,\theta_x, root\,\theta_y, root\,\theta_z}_{\text{root rot}}, \underbrace{lknee\,\theta_x, lknee\,\theta_y}, \dots]$$

all values in local coordinates

$\boxed{EX}$ Kinematic Chain




Goal: Compute the global positions of $p_1, p_2, p_3$ given the local transforms at $p_1$ $p_2$ $p_3$, e.g. $T_1^0$, $T_2^1$, $T_3^2$

Approach: Compute $T_1^0$, $T_2^0$, and $T_3^0$ to get the global positions of $p_1$ $p_2$ & $p_3$.

$p_1$ wrt to $F_0$

$$p_1^0 = T_1^0 p_1^1, \quad \text{where } p_1^1 = (0,0,0,1)^T$$

$$p_2^0 = T_2^0 p_2^2 = T_1^0 T_2^1 p_2^2 \leftarrow p_2^2 = (0,0,0,1)^T$$

$$p_3^0 = T_3^0 p_3^3 = T_1^0 T_2^1 T_3^2 p_3^3$$

$\boxed{EX}$ Suppose

$$d_1^0 = (1,-1,1)^T \qquad R_1^0 = R_z(-45)$$
$$d_2^1 = (2,0,0)^T \qquad R_2^1 = R_z(-30)$$
$$d_3^2 = (2,-1,0)^T \qquad R_3^2 = I$$

$$p_3^0 = T_3^0 p_3^3 = T_1^0 T_2^1 T_3^2 p_3^3$$

$$= \left[\begin{array}{c|c} R_1^0 & d_1^0 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} R_2^1 & d_2^1 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} R_3^2 & d_3^2 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c} \phi \\ \hline 1 \end{array}\right]$$

$\phi$ is a $3 \times 1$ vector

$$= \left[\begin{array}{c|c} R_1 & d_1 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} R_2 & d_2 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} R_3 & d_3 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c} \\ \hline 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} R_1^0 R_2^1 & R_1^0 d_2^1 + d_1^0 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c|c} R_3^2 & d_3^2 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c} \emptyset \\ \hline 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} R_1^0 R_2^1 R_3^2 & R_1^0 R_2^1 d_3^2 + R_1^0 d_2^1 + d_1^0 \\ \hline 0 & 1 \end{array}\right]\left[\begin{array}{c} \emptyset \\ \hline 1 \end{array}\right]$$

= do later

<u>Forward Kinematics Alg</u> (FK): generalizes process we just did w/ the kinematic chain to any skeleton

<u>FK</u>: Recursively compute local2global $F_j^0$ for each joint $j$

<u>Idea</u>: Do a tree traversal on each joint. Start at root

```
Joint::fk()
  if (mParent != NULL)
     local2global = (mParent -> local2global) * local2parent
  else
     local2global = local2parent;   //root

  for each child
     child -> fk()
```

$$\text{local2global} = \underbrace{(\text{mParent} \to \text{local2global})}_{F_i^0} * \underbrace{\text{local2parent}}_{F_j^i}$$