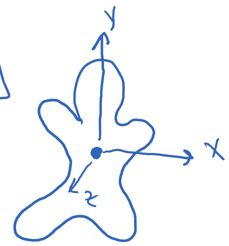
Rotations

Tuesday, September 28, 2021 9:40 AM

Goal: Represent "which way" an object is facing
→ how? using directions of 3x3 matrices

EX



Y is UP
Z is FORWARD
(points out of page)

The direction of the character's Z direction is the direction it is facing

The rotation matrix that represents the orientation of character corresponds to the matrix $R = \left[X \mid Y \mid Z \right]$ ← 3x3 matrix

EX If the character is facing $Z = (\sqrt{2}/2, 0, \sqrt{2}/2)$, then $X = \left(\frac{\sqrt{2}}{2}, 0, \frac{-\sqrt{2}}{2}\right)$, $Y = (0, 1, 0)$

$$R = \begin{array}{c} \\ \\ \\ \\ \end{array}\left[\begin{array}{c|c|c} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{array}\right]$$
$$\begin{array}{ccc} \quad\quad X & Y & Z \end{array}$$

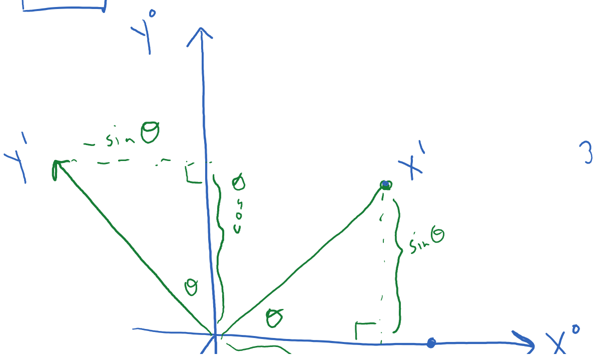
Note: the X, Y, Z directions of the character are all perpendicular to each other

Rotation Matrices:

Perspective: A rotation matrix maps from one coordinate system to another

note: $c\theta = \cos(\theta)$
$s\theta = \sin(\theta)$

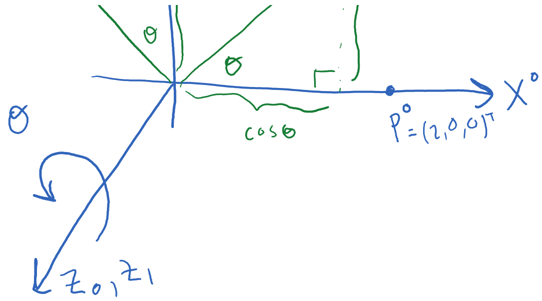
EX Rotation around Z



$$R_z(\theta) = \left(\begin{array}{c|c|c} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{array}\right)$$

↑ 3x3 rotation around z by angle θ

EX $R_z(90) = \left(\begin{array}{c|c|c} 0 & -1 & 0 \\ \ldots & 0 & 0 \end{array}\right.$



$$y' = \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} \qquad x' = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

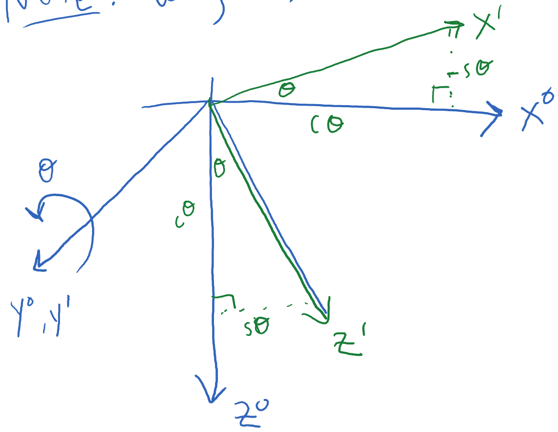
$\boxed{EX}$ $R_z(90) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$R_z(90)\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{pmatrix}$$ ← rotation around y axis

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{pmatrix}$$

__NOTE__: why is Y matrix has the negative $-s\theta$ in bottom, first col?



$$x' = (c\theta, 0, -s\theta)$$
$$y' = (s\theta, 0, c\theta)$$

Properties of rotation matrices:

① Orthonormal

→ each column has length 1

→ each column is perpendicular to the others

→ inverse is equal to the transpose, e.g.

if R is orthonormal, then $R^{-1} = R^T$

$\boxed{EX}$ Show that $R_z(\theta)\ R_z(\theta)^T = I$

if R is orthonormal ...

$\boxed{EX}$ Show that $R_z(\theta)\, R_z(\theta)^T = I$

$$\begin{pmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c^2\theta + s^2\theta & \cancel{c\theta s\theta - c\theta s\theta} & 0 \\ \cancel{s\theta c\theta - c\theta s\theta} & c^2\theta + s^2\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{EX}$ Show that the first 2 columns of $R_z(\theta)$ are perpendicular.
Show the dot product is zero

Aside: $\cos\theta = \dfrac{a \cdot b}{\|a\|\|b\|}$. $\cos(90) = 0$

$$(c\theta \;\; s\theta \;\; 0) \cdot (-s\theta \;\; c\theta \;\; 0) = -c\theta s\theta + s\theta c\theta + 0 = 0 \quad \checkmark$$

Euler Angles

Defn: An euler angle is a triplet (α, θ, β) of angles to rotate around each local object axes