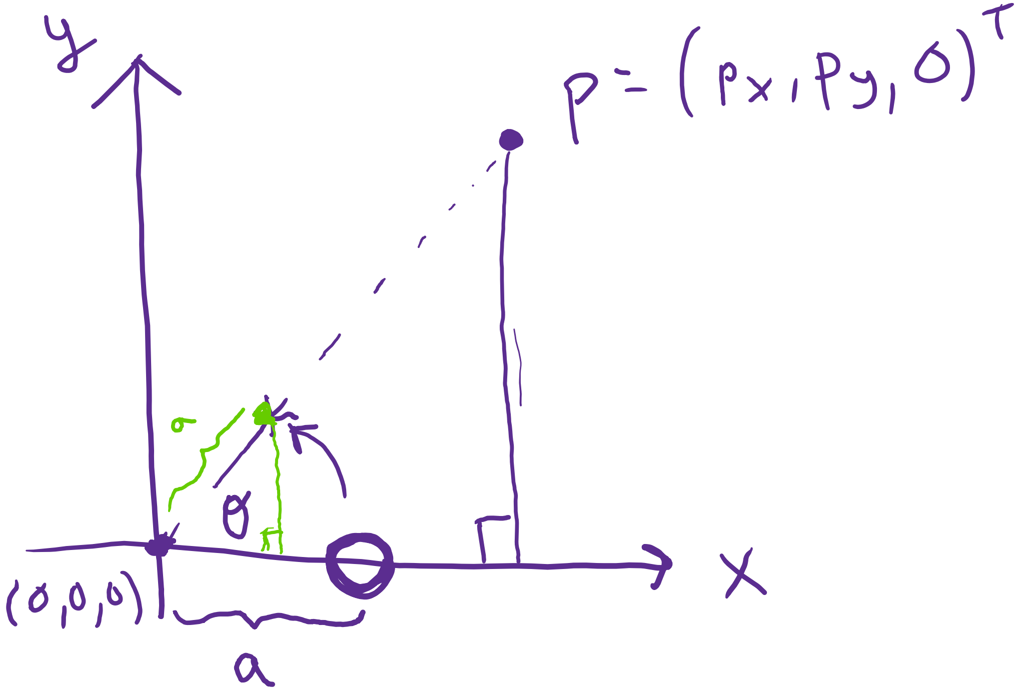
Tangent continued

$\boxed{\text{EX}}$ Using tan to rotate in a desired direction

Suppose we have target p. How can we rotate towards it?



Let's define a sphere at position $(a, 0, 0)^T$

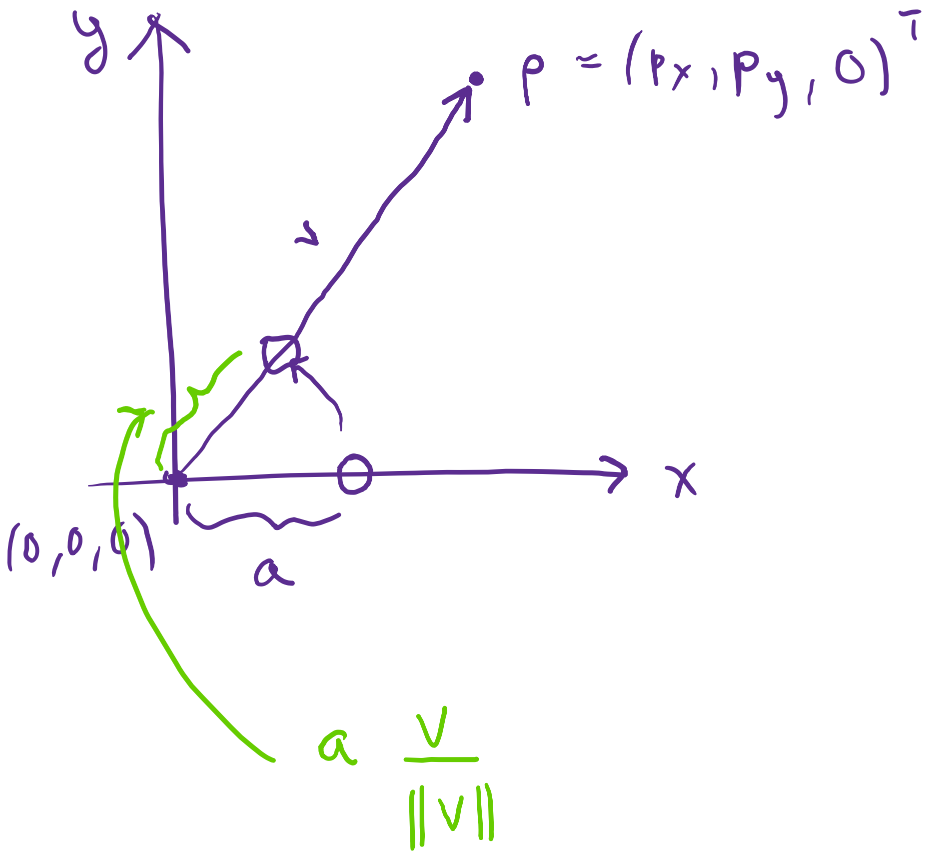
a is the desired distance from the <u>pivot point</u>

<u>Step 1</u>: Find $\theta = \text{atan2}(p_y, p_x)$

<u>Step 2</u>: Compute rotated Sphere Pos

$$\text{spherePos} = \begin{pmatrix} a\cos\theta \\ a\sin\theta \\ 0 \end{pmatrix}$$

Alternate Approach



① Compute the direction between p & the origin

Let $v = p - \bar{o} = p$

② Normalize v to have unit length

$$\frac{v}{\|v\|}$$

③ Scale the unit direction to desired distance a

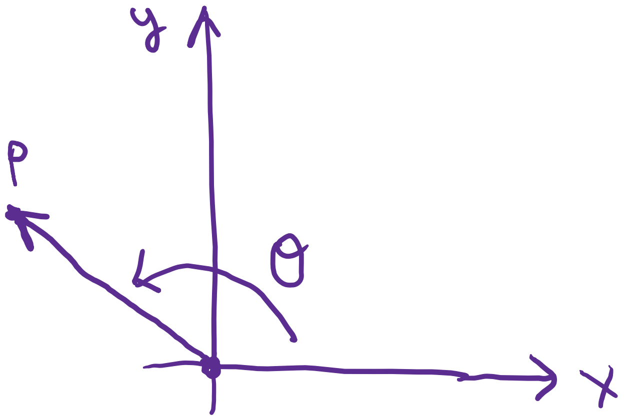
$$a \frac{v}{\|v\|}$$

Example

Suppose $p = (-5, 5, 0)^T$ and $\underline{a = 2}$. Compute the position of our sphere so that it rotates around $(0,0,0)^T$

(a) Using atan2

(b) Using a direction vector



(a) Compute $\theta = \text{atan2}(5, -5) = 135°$

$\therefore \text{spherePos} = 2 \begin{pmatrix} \cos(135) \\ \sin(135) \\ 0 \end{pmatrix} = 2 \begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ \sqrt{2} \\ 0 \end{pmatrix}$

(b)

Example: Method 2

(b) Need to compute spherePos $= a\left(\frac{p}{\|p\|}\right) = \frac{a}{\|p\|}\,\mathbf{p}$

$$\|p\| = \sqrt{(-5)^2 + 5^2 + 0^2} = \sqrt{25+25} = \sqrt{2 \cdot 25} = 5\sqrt{2}$$

$$\therefore \text{spherePos} = \frac{2}{5\sqrt{2}}\begin{pmatrix}-5\\5\\0\end{pmatrix} = \frac{\cancel{2}\sqrt{2}}{5\cdot\cancel{2}}\begin{pmatrix}-5\\5\\0\end{pmatrix} = \sqrt{2}\begin{pmatrix}-1\\1\\0\end{pmatrix} = \begin{pmatrix}-\sqrt{2}\\\sqrt{2}\\0\end{pmatrix}$$

What if we rotate around a pivot point other than the origin?

Method 1 : Step 1 : Subtract pivot pt c from p : $p' = p - c$

$\theta = \text{atan2}(p'_y, p'_x)$

Step 2 : Compute $p'' = a \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$

Step 3 : Add pivot pt back : $p'' + c$

Method 2 : Modify eqn to be $a \left(\dfrac{p - c}{\|p - c\|} \right)$

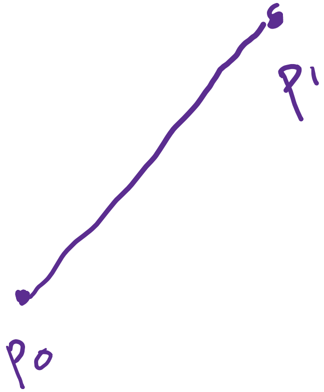
Methods of Interpolation

Idea: Given two values p_0 & p_1, interpolation generates intermediate values

→ aka tweening, easings

Linear:

Idea: Use a line to fill intermediate values



$$p(t) = p_0 + t(p_1 - p_0), \quad t \in [0,1]$$

$$= p_0 + tp_1 - tp_0$$

$$= p_0(1-t) + p_1 t$$

Two Perspectives:

Geometric (Line Eqn)

Weighted Sum:

coefficients sum to 1

coefficients $\in [0,1]$

Implementation

Design an alg. that animates to color of a sphere from red to green over one second.

setup()

```
red = vec3(1,0,0)
green = vec3(0,1,0)
t=0
```

data

```
vec3 red;
vec3 green;
float t;
```

scene()

```
t += dt()
t = clamp(t, 0, 1) // t = min(t, 1);
vec3 c = red (1-t) + green * t
setColor(c)
drawSphere(vec3(0), 100);
```

What if we want to control the duration of the interpolation?

Suppose, we want the transition to occur over T seconds?

Idea: Let's define a normalized interpolation value u

→ when elapsedTime $= 0$, $u = 0$

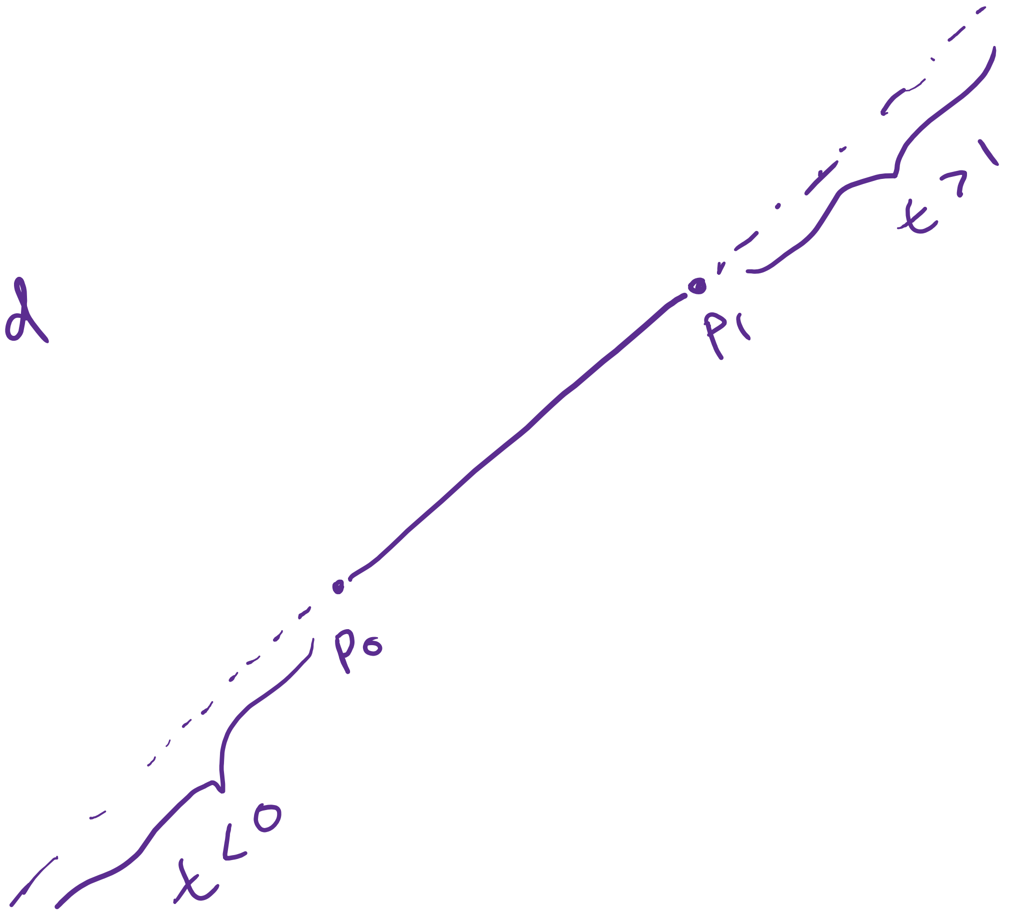
→ when elapsedTime $= T$, $u = 1$

$$\Rightarrow \quad u = \frac{\text{elapsedTime}}{T}$$

What happens when t in not in the range [0,1]?

Called <u>extrapolation</u>

Results are undefined



Example

Suppose we wish to linearly interpolate from position $p_0 = (1,0,0)^T$ to $p_1 = (0,0,1)^T$ in 8 seconds.

[This is an example of key framing:

key $\emptyset$ = $\langle 0, p_0 \rangle$, key 1 = $\langle 8, p_1 \rangle$

time value

What is the position at $t = 3s$?

① Compute $u = t/T = 3/8$

② Compute $p(t) = (1,0,0)^T (1 - 3/8) + (0,0,1)^T (3/8)$

$= (5/8, 0, 3/8)^T$

Cubic Interpolation

Idea: Use a degree-3 poly to interpolate

$t \in [0,1]$

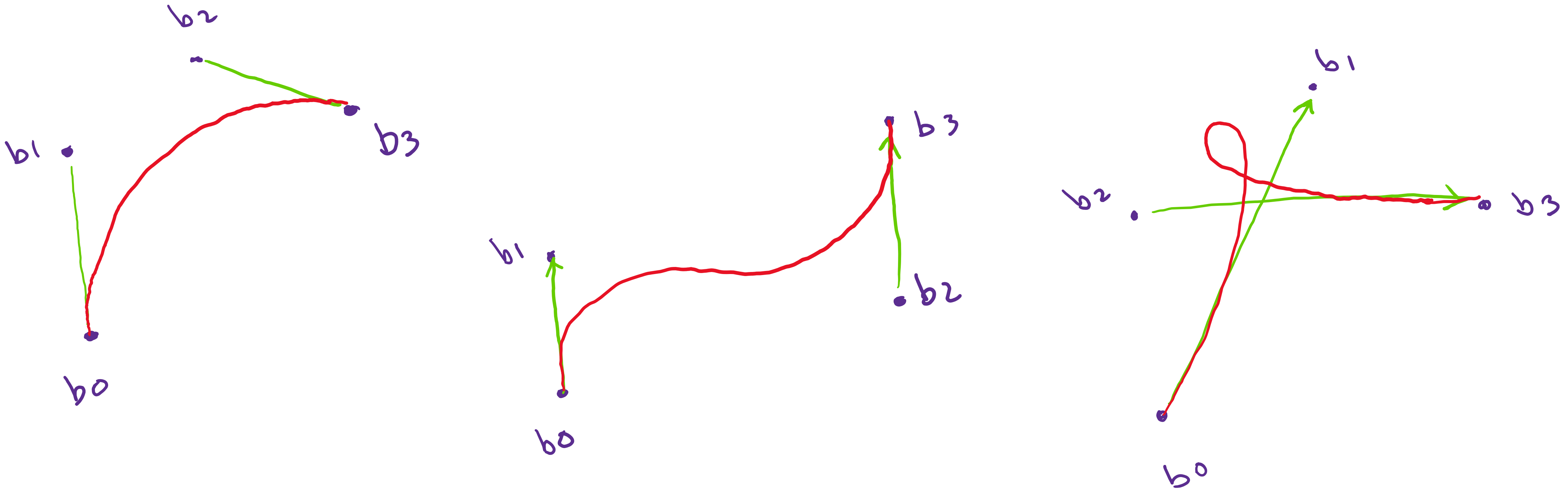
$$p(t) = (1-t)^3 b_0 + 3t(1-t)^2 b_1 + 3t^2(1-t) b_2 + t^3 b_3$$

b_0 & b_3 are our keys,

b_i's are called control points,

("control" shape of the curve)

Cubic Interpolation



Example

$$p(t) = (1-t)^3 b_0 + 3t(1-t)^2 b_1 + 3t^2(1-t) b_2 + t^3 b_3$$

what is the value of this curve

when $t = 0$? $p(0) = 1^3 b_0 = b_0$

when $t = 1$? $p(1) = 1^3 b_3 = b_3$

Exercise: What is the sum of the coefficients of p(t) for our cubic polynomial

$$(1-t)^3 + 3t(1-t)^2 + 3t^2(1-t) + t^3$$

$$= (1-3t+3t^2-t^3) + 3t(1-2t+t^2) + 3t^2(1-t) + t^3$$

$$= 1 - \cancel{3t} + 3t^2 - \cancel{t^3} + \cancel{3t} - 6t^2 + \cancel{3t^3} + 3t^2 - \cancel{3t^3} + \cancel{t^3}$$

$$= 1 + \cancel{6t^2} - \cancel{6t^2}$$

$$= 1$$